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The ^{30}Mg$(t,p)^{32}$Mg "puzzle" reexamined

A. O. Macchiavelli, H. L. Crawford, C. M. Campbell, R. M. Clark, M. Cromaz, P. Fallon, M. D. Jones, I. Y. Lee, and M. Salathe
Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA

B. A. Brown
Department of Physics and Astronomy and National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824, USA

A. Poves
Departamento de Fisica Teorica and IFT-UAM/CSIC, Universidad Autonoma de Madrid, E-28049 Madrid, Spain

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Background: Competing interpretations of the results of a ^{30}Mg$(t,p)^{32}$Mg measurement populating the ground state and 0_2^+ state in ^{32}Mg, both limited to a two-state mixing description, have left an open question regarding the nature of the ^{32}Mg ground state.
Purpose: Inspired by recent shell-model calculations, we explore the possibility of a consistent interpretation of the available data for the low-lying 0^+ states in ^{32}Mg by expanding the description from two-level to three-level mixing.
Methods: A phenomenological three-level mixing model of unperturbed 0p0h, 2p2h, and 4p4h states is applied to describe both the excitation energies in ^{32}Mg and the transfer reaction cross sections.
Results: Within this approach, self-consistent solutions exist that provide good agreement with the available experimental information obtained from the ^{30}Mg$(t,p)^{32}$Mg reaction.
Conclusion: The inclusion of the third state, namely the 4p4h configuration, resolves the "puzzle" that results from a two-levelmodel interpretation of the same data. In our analysis, the ^{32}Mg ground state emerges naturally as dominated by intruder (2p2h and 4p4h) configurations, at the 95% level.

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Introduction. The $N = 20$ island of inversion has been the subject of intense work, both experimentally and theoretically [1]. As protons are removed from $^{40}_{20}$Ca, changes in the balance between the monopole shifts of the single-particle levels and the pairing plus quadrupole correlations erode the $N = 20$ shell gap, leading to deformed (2p2h, 4p4h) ground states in these nuclei, expected *a priori* to be semimagic and spherical. The nucleus ^{32}Mg takes center stage in this region, where neutron pairs promoted from sd to fp levels across the narrowed $N = 20$ gap are energetically favored. The enhanced occupation of these deformation-driving fp orbitals causes the nucleus to deform [2,3].

Wimmer *et al.* [4] studied the two-neutron transfer reaction ^{30}Mg$(t,p)^{32}$Mg at CERN/ISOLDE and discovered the first excited 0_2^+ state (at 1.058 MeV), which was attributed to be largely the 0p0h spherical state. Following on these results, Fortune [5] carried out a two-level model analysis of the reaction data and put forward the puzzling conclusion that in ^{32}Mg it is the 0_1^+ ground state which is actually dominated by the sd-shell components (≈80%) and the excited 0_2^+ by the fp-shell 2p2h intruder configuration, contrary to the accepted interpretation (see also Ref. [6]).

Large-scale shell-model calculations [7], however, predict the coexistence of 0p0h, 2p2h, and 4p4h states in the low-lying excitation spectra of the $N \sim 20$ Mg nuclei, calling into question the validity of a two-level approach. Inspired by these results, we have revisited the analysis of Fortune [5,8], extending it now to a three-level mixing.

The approach. To investigate the validity of a three-state mixing model to describe the low-energy structure in ^{32}Mg, and in particular to explain the observations of the ^{30}Mg$(t,p)^{32}$Mg measurement [4], without the complexities of a full large-scale shell-model calculation, we assume mixing between unperturbed, pure $|npn\text{h}\rangle$ configurations, where $n =$ 0, 2, and 4. The mixing matrix is tridiagonal, and we make the simplifying assumption that the interaction strengths between the $|0\text{p}0\text{h}\rangle$ and $|2\text{p}2\text{h}\rangle$ configurations and the $|2\text{p}2\text{h}\rangle$ and $|4\text{p}4\text{h}\rangle$ configurations are equal ($-V$). Thus, the mixing matrix has the form

$$\begin{pmatrix} e_0 & -V & 0 \\ -V & e_2 & -V \\ 0 & -V & e_4 \end{pmatrix}, \quad (1)$$

where e_0, e_2, and e_4 are the energies of the unperturbed 0p0h, 2p2h, and 4p4h configurations respectively. At this point it is important to comment that while one would be tempted to think that this could be equivalent to the mixing of two states, namely a spherical one (0p0h configuration) and a deformed one (a combination of 2p2h and 4p4h configurations), the 3×3 matrix above does not reduce to a 1×1 plus 2×2 block sub-matrices. However, by taking the limit of the 4p4h at very high energy, we do recover a 2×2 matrix and reproduce Fortune's results.

From the diagonalization we obtain wave functions of the form

$$|0_j^+\rangle = \alpha_j|0\text{p}0\text{h}\rangle + \beta_j|2\text{p}2\text{h}\rangle + \gamma_j|4\text{p}4\text{h}\rangle, \quad (2)$$

where α_j, β_j, and γ_j are constrained by the normalization condition (i.e. $\alpha_j^2 + \beta_j^2 + \gamma_j^2 = 1$).

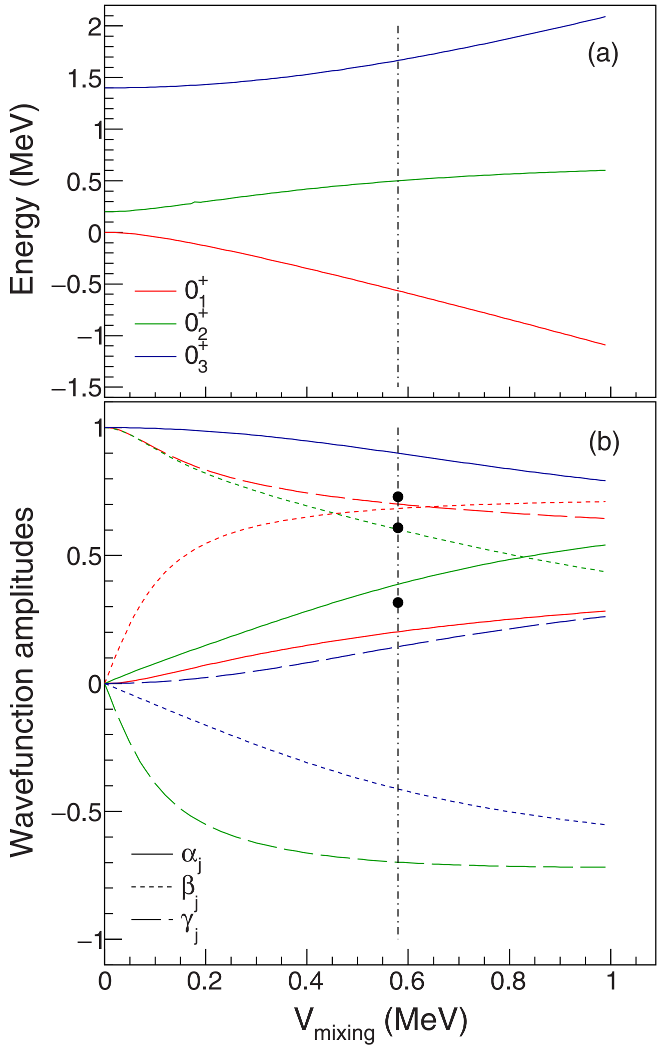

FIG. 1. Graphical illustration of the solutions for diagonalization of the matrix of Eq. (1) as a function of V. The top panel (a) shows the evolution of the energy levels for the mixed 0^+ states. The dot-dashed line at $V = 0.58$ MeV represents the interaction strength at which the excitation energy for 0_2^+ equals the experimental value. The bottom panel (b) shows the evolution of the wave-function amplitudes for the three 0^+ states.

As a starting point we take the unperturbed values, based on the large-scale shell-model calculations of Ref. [7], to be $e_0 = 1.4$ MeV, $e_2 = 0.2$ MeV, and $e_4 = 0$ MeV.

Calculations and results. Diagonalizing the matrix of Eq. (1), we directly obtain the mixed state eigenvalues as a function of the mixing matrix element V—these results are plotted in Fig. 1(a), for the ground state 0_1^+ (red), first excited state 0_2^+ (green), and second excited state 0_3^+ (blue). The corresponding wave-function coefficients for these states are plotted, with the same color coding in Fig. 1(b), where α coefficients for the $|0p0h\rangle$ contribution are the solid lines, β coefficients, for the $|2p2h\rangle$ component, are dotted lines, and the γ $|4p4h\rangle$ coefficients are dashed lines.

Considering the energies of the mixed 0^+ states, we can constrain the mixing strength V according to the experimentally observed separation between the 0_1^+ and 0_2^+ states of 1.06 MeV [4]. This energy separation occurs for a mixing strength of $V = 0.58$ MeV. For this mixing strength, the corresponding energies and wave-function amplitudes for the three 0^+ states are summarized in Table I. In particular for the ground state we have $\alpha_1 = 0.20, \beta_1 = 0.68$, and $\gamma_1 = 0.70$. This corresponds well to the amplitudes obtained for the ground state of ^{32}Mg using a full large-scale shell-model calculation with the SDPF-U-MIX effective interaction, where $\alpha = 0.32$, $\beta = 0.72$, and $\gamma = 0.61$ [9]. These shell-model based amplitudes are plotted as the black circles in Fig. 1.

TABLE I. Results for the wave-function amplitudes and energies of the first three 0^+ states obtained from diagonalization of Eq. (1) for V = 0.58 MeV.

State	Excitation energy [MeV]	α $\|0p0h\rangle$	β $\|2p2h\rangle$	γ $\|4p4h\rangle$
0_1^+	0.0	0.20	0.68	0.70
0_2^+	1.06	0.39	0.60	−0.70
0_3^+	2.22	0.90	−0.41	0.14

Additional support for our approach can be assessed by looking at the overlaps of the three lowest 0^+ states obtained within the full shell-model space with the lowest energy states of the three pure $npnh$ configurations. These are shown in Table II. As seen, over 90% of the full wave functions come indeed from the pure configurations, which justifies the truncation to a three-level model.[1]

To make further ties with experiment and check the consistency of this simple model, we adopt a description for the ^{30}Mg ground state as

$$|0_1^+(^{30}\mathrm{Mg})\rangle = \epsilon|0p0h\rangle + \sqrt{1-\epsilon^2}|2p2h\rangle, \tag{3}$$

under the assumption that a significant 4p4h contribution in the ground state of ^{30}Mg is very unlikely, as this is expected to lie at much higher energy. With this description, we can calculate the cross sections for two-neutron transfer to the ground state and excited 0^+ states in ^{32}Mg from the ground state of ^{30}Mg. The cross section to the ith 0^+ state in ^{32}Mg from the ^{30}Mg ground state can be expressed as

$$\sigma_{0_i^+} \propto (\epsilon\alpha_i T_{0,0} + \epsilon\beta_i T_{0,2} + \sqrt{1-\epsilon^2}\beta_i T_{2,2} + \sqrt{1-\epsilon^2}\gamma_i T_{2,4})^2, \tag{4}$$

where $T_{a,b}$ is the two-nucleon transfer amplitude between the $|apah\rangle$ state in ^{30}Mg and the $|bpbh\rangle$ component of the wave function in ^{32}Mg. Referring to the schematic diagram shown in Fig. 2, we note that the two-neutron transfer does not connect the ^{30}Mg $|0p0h\rangle$ wavefunction component with the ^{32}Mg $|4p4h\rangle$, nor the ^{30}Mg $|2p2h\rangle$ contribution with the ^{32}Mg $|0p0h\rangle$ wavefunction, i.e., $T_{0,4}$ and $T_{2,0} = 0$. We can further realize that the amplitudes $T_{0,0}$ and $T_{2,2}$ both correspond

[1]The initial energies of the 3×3 model that approximately reproduce these percentages for $V = 0.6$ are $e_0 = 0.69$, $e_2 = 0.0$, and $e_4 = 0.15$ MeV, corresponding to unperturbed values derived from the overlaps in Table II.

TABLE II. The overlaps (in percent) of the three lowest 0^+ states in the full shell-model space with the lowest energy states of the three pure *npn*h configurations.

Full/pure	0p0h	2p2h	4p4h	Sum
1	10	53	31	94
2	32	8	53	93
3	48	31	12	91

to the addition of a pair of *sd* neutrons (T_{sd}), while $T_{0,2}$ and $T_{2,4}$ relate to the addition of a pair of *fp* neutrons (T_{fp}), and make the simplifying assumption that $T_{0,0} = T_{2,2}$ and $T_{0,2} = T_{2,4}$. This is supported if one considers the transfer amplitudes in a single-particle description, which can be estimated with simple coefficients of fractional parentage and the overlap of two neutrons in the triton with the orbitals of the target nucleus [10]. In line with the arguments of Fortune [5] we further assume that $T_{fp} = R \times T_{sd}$, and used his value of $R = 2$, also validated in our approach by the experimental constraint on the total cross section populating the 0_1^+, 0_2^+, and 0_3^+ states, which is 17.0(9) mb [4]. We also note that a variation of R within a factor of two (up or down) does not alter the results of the calculations described below.

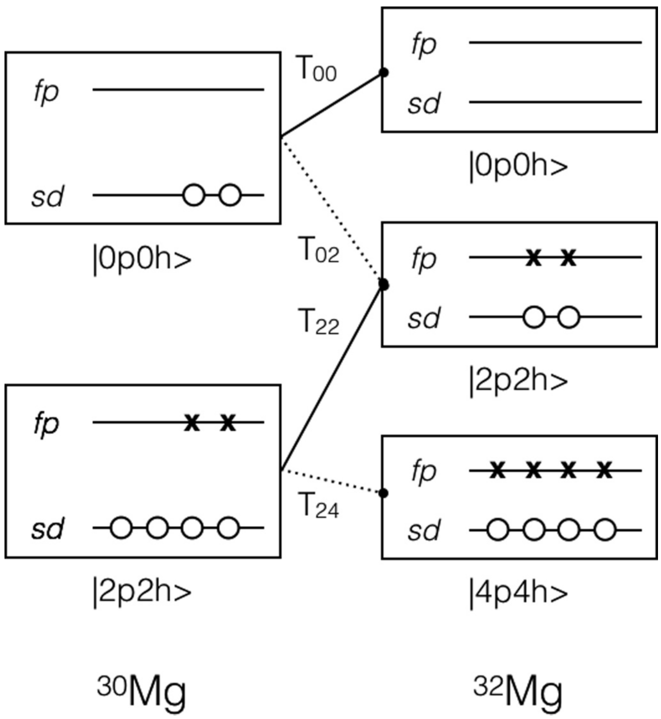

FIG. 2. Schematic representation of the two-neutron transfers under consideration. The components of the ^{30}Mg ground-state wave function connect to components of the ^{32}Mg 0^+ wave functions as shown. The solid connecting lines correspond to transfer of *sd* shell neutrons, while the dotted connecting lines correspond to transfer of a pair of *fp* shell neutrons. Open circles represent holes in the *sd* shell, while the crosses represent particles in the *fp* shell.

A cross-section ratio equation can then be defined, which simplifies to

$$\frac{\sigma_{0_i^+}}{\sigma_{0_j^+}} = \left[\frac{\epsilon(\alpha_i + \beta_i R) + \sqrt{1-\epsilon^2}(\beta_i + \gamma_i R)}{\epsilon(\alpha_j + \beta_j R) + \sqrt{1-\epsilon^2}(\beta_j + \gamma_j R)}\right]^2. \quad (5)$$

We evaluate the cross-section ratio $\sigma_{0_2^+}/\sigma_{0_1^+}$ directly as a function of V and ϵ, and obtain the results plotted in Fig. 3. Experimentally, this cross-section ratio has a value $\sigma_{0_2^+}/\sigma_{0_1^+} = 0.62(6)$ [4]. Taking our earlier result constrained by the experimental excitation energy of $E(0_2^+) = 1.06$ MeV for $V = 0.58$ MeV [Fig. 1(a), dot-dashed line], the experimental ratio is reproduced for a ^{30}Mg ground-state wave function:

$$|0_1^+(^{30}\text{Mg})\rangle = 0.99|0\text{p}0\text{h}\rangle + 0.17|2\text{p}2\text{h}\rangle. \quad (6)$$

The amplitudes are in good agreement with those deduced from the $E0$ transition strength in ^{30}Mg [11], where $\epsilon = 0.983(15)$.

At this point it is relevant to comment on the possible effect of the Q-value dependence of the transfer amplitudes. We have considered this effect by looking at the results of a distorted wave Born approximation (DWBA) calculation using the code DWUCK4 [12]. As could be expected, the transfer to the excited 0_2^+ state is suppressed more than the ground state, but the conclusions above do not change much. The effect can be cast

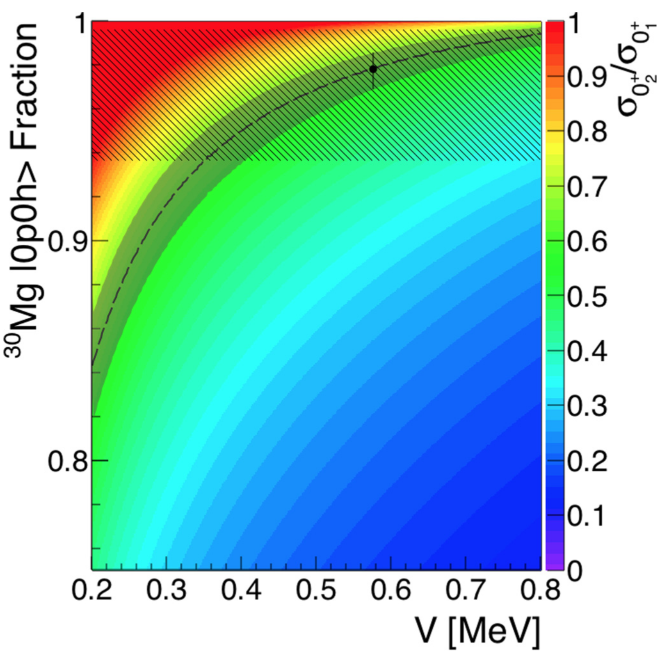

FIG. 3. Cross-section ratio $\sigma_{0_2^+}/\sigma_{0_1^+}$ calculated according to Eq. (5) based on the ^{32}Mg wave-function amplitudes as plotted in Fig. 1(b) as a function of V and the $|0\text{p}0\text{h}\rangle$ squared amplitude in the ^{30}Mg ground state [ϵ^2 in Eq. (3)]. The hashed area represents the results of Ref. [11], while the dashed line and shaded error band indicate the experimental range of the cross-section ratio. The black data point indicates the mixing strength, V, which reproduces the experimental $E(0_2^+)$.

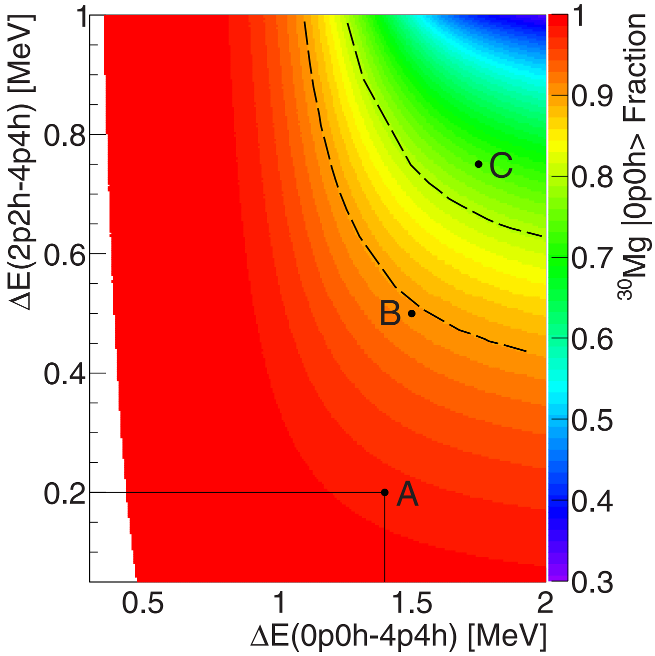

FIG. 4. Percent squared amplitude of the 0p0h component in the ^{30}Mg ground state [ϵ^2 in Eq. (3)] required to explain the experimental data as a function of the energies of the unperturbed states. Point A shows the solution [Eq. (6)] with the energies fixed at the unperturbed values discussed in Sec. II. The dashed lines indicate possible solutions with a 0p0h amplitude for the ^{30}Mg ground state at 80% and 90%. As discussed in the text, points B and C are used as a reference with respect to the ^{32}Mg wave functions.

in the form of a correction to the experimental cross-section ratio, making it larger. As seen in Fig. 3, this will correspond to a solution with ^{30}Mg closer to a spherical shape.

Within our model we find the ratio $\sigma_{0_3^+}/\sigma_{0_1^+}$ to be very small and consistent with the non-observation of a third 0^+ state in the work of Wimmer *et al.* [4]. To obtain a summed cross-section of $(\sigma_{0_1^+}+\sigma_{0_2^+}+\sigma_{0_3^+}) = 17.0(9)$ mb, the required normalization is $\sigma_{sd} = 3.1$ mb, in line with the expectations for pure *sd* neutron-pair transfer [12].

Finally, while the wave function above for the ^{30}Mg ground state agrees well with the results of Ref. [11] there is some model dependence in that analysis arising from the $E0$ matrix elements used. It is possible that an equally realistic description for the structure of ^{30}Mg may be consistent with a lower 0p0h amplitude, down to ≈80%. We explore the existence of other solutions within our model framework, in terms of the energies of the unperturbed initial states: e_0, e_2, and e_4. As seen in Fig. 4 modest changes in the initial energies allow for physically sound solutions, which reproduce the experimental cross-section ratio and $E(0_2^+)$ in ^{32}Mg, and are more consistent with a ^{30}Mg ground state less strictly dominated by a 0p0h configuration (i.e., the region between the dashed lines in the figure). These changes, in particular the energy of the 4p4h configuration, could be explained by small adjustments of the single-particle monopole shifts and the quadrupole interaction.

TABLE III. Results for the mixing strength required to reproduce the experimental $E(0_2^+)$ and the resulting wave-function amplitudes in ^{32}Mg corresponding to the solutions indicated by points A, B, and C in Fig. 4 with (e_0,e_2,e_4) [MeV] = (1.4, 0.2, 0.0), (1.5, 0.5, 0.0), and (1.75, 0.75, 0.0) respectively.

State	Point	V [MeV]	α $\lvert 0p0h\rangle$	β $\lvert 2p2h\rangle$	γ $\lvert 4p4h\rangle$
0_1^+			0.20	0.68	0.70
0_2^+	A	0.576	0.39	0.60	−0.70
0_3^+			0.90	−0.41	0.14
0_1^+			0.17	0.59	0.79
0_2^+	B	0.566	0.44	0.67	−0.60
0_3^+			0.88	−0.45	0.14
0_1^+			0.11	0.47	0.88
0_2^+	C	0.482	0.40	0.79	−0.48
0_3^+			0.91	−0.40	0.10

It is also of interest to compare the wave functions for the lowest $0_{1,2}^+$ states in ^{32}Mg obtained for solution A, with those obtained for solutions B and C, listed in Table III. Solutions B and C appear quite different from A in the figure, i.e., in the ^{30}Mg ground-state wave function obtained, but the amplitudes of the ^{32}Mg wave functions are in fact quite consistent and robustly confirm its ground state as dominated by intruder 2p2h and 4p4h excitations.

Conclusion. It is clear that the inclusion of the third state, namely the 4p4h configuration, resolves the "puzzle" of ^{32}Mg proposed by Fortune [5,8], and the ^{32}Mg ground state emerges naturally as dominated at the 95% level by intruder (2p2h or 4p4h) configurations. Within a simple three-level model, self-consistent solutions exist that provide good agreement with the experimental excitation energy of the 0_2^+ state, the cross-section ratio $\sigma_{0_2^+}/\sigma_{0_1^+}$, and the summed cross section $(\sigma_{0_1^+}+\sigma_{0_2^+}+\sigma_{0_3^+})$ with a reasonable value for the cross section for transfer of two *sd* neutrons. These scenarios also indicate a ^{30}Mg ground state dominated by the 0p0h component, in line with experimental evidence and shell-model expectations. While further experimental information, such as the lifetime of the 0_2^+ state, will further constrain shell models and more complex and complete descriptions of nuclear structure in this region, the ^{32}Mg "puzzle" is, to first order, resolved.

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